

# Finite State Verification

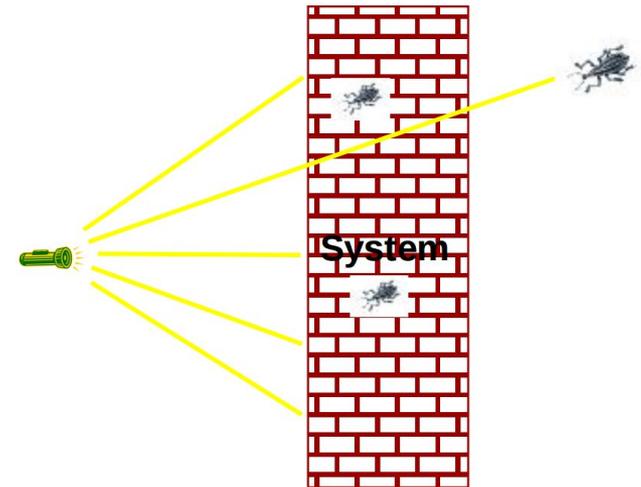
CSCE 747 - Lecture 21 - 03/28/2017

# So, You Want to Perform Verification...

- You have a property that you want your program to obey.
- Great! Let's write some tests!
- **Does testing guarantee that the requirement is met?**
  - Not quite...
    - Testing can make a **statistical** argument in favor of verification, but usually cannot guarantee that the requirement holds in *all* situations.

# Testing

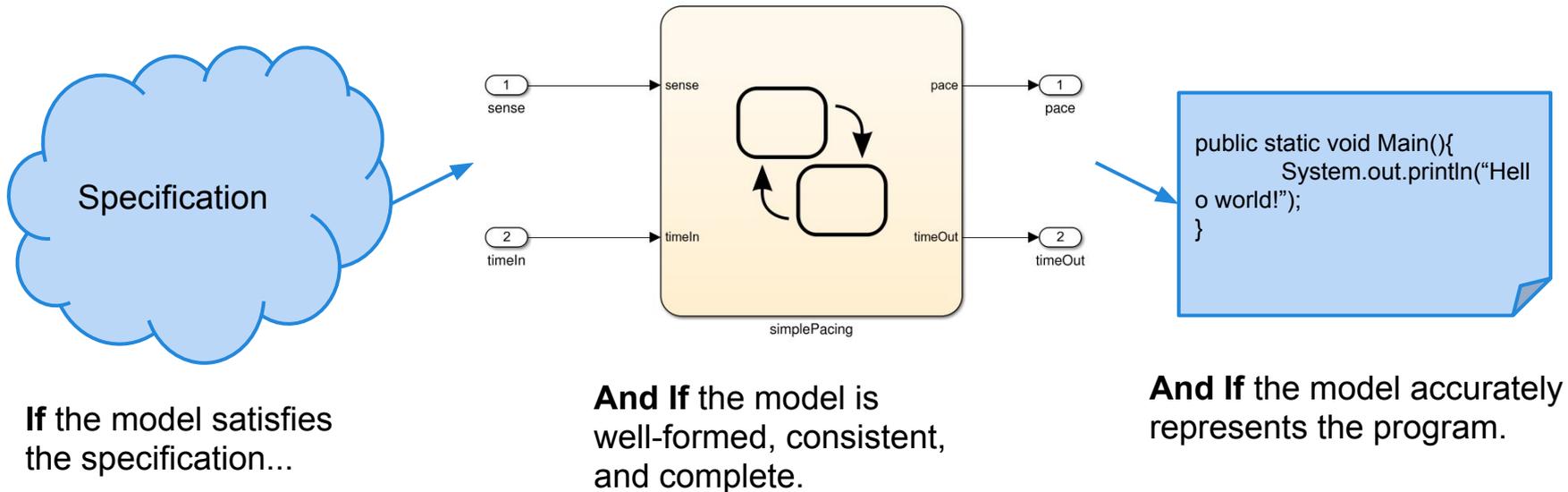
- Any real system has a near-infinite number of possible inputs.
- Some faults trigger failures extremely rarely, or under conditions that are hard to control and recreate through testing.
- How can we *prove* that our system meets the property?



# What About a Model?

- We have previously used models to analyze programs, and to generate test cases.
- Models can be used to “tame” the complexity of the program.
  - Models are simpler than the real program.
  - By abstracting away unnecessary details, we can learn important insights.
- Perhaps models can be used to verify the full programs!

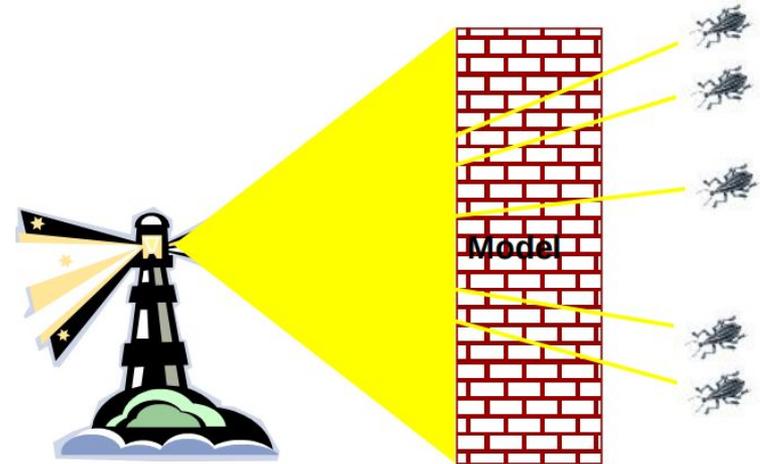
# What Can We Do With This Model?



If we can show that the model satisfies the requirement, then the program should as well.

# Finite-State Verification

- Express specification as a set of logical properties, written as Boolean formulae.
- Exhaustively search the state space of the model for violations of those properties.
- If the property holds - proof that the model is correct.
- Contrast with testing - no violation might just mean bad tests.



# Today's Goals

- Formulating specification statements as formal logical expressions.
  - Introduction to temporal logic.
- Building behavioral models in NuSMV.
- Performing finite-state verification over the model.
  - Exhaustive search algorithms.

# Expressing Specification Statements as Provable Properties

# Expressing Properties

- Properties expressed in a formal logic.
  - Temporal logic ensures that properties hold over execution paths, not just at a single point in time.
- Safety Properties
  - System **never** reaches bad state.
  - **Always** in some good state.
- Liveness Properties
  - **Eventually** useful things happen.
  - **Fairness** criteria.

# Temporal Logic

- Sets of rules and symbolism for representing propositions qualified over time.
- Linear Time Logic (LTL)
  - Reason about events over a timeline.
- Computation Tree Logic (CTL)
  - Branching logic that can reason about multiple timelines.
- We need both forms of logic - each can express properties that the other cannot.

# Linear Time Logic Formulae

Formulae written with propositional variables (boolean properties), logical operators (and, or, not, implication), and a set of modal operators:

<b>X (next)</b>	X hunger	In the next state, I will be hungry.
<b>G (globally)</b>	G hunger	In all future states, I will be hungry.
<b>F (finally)</b>	F hunger	Eventually, there will be a state where I am hungry.
<b>U (until)</b>	hunger U burger	I will be hungry until I start to eat a burger.
<b>R (release)</b>	hunger R burger	I will cease to be hungry after I eat a burger.

# LTL Examples

- **X (next)** - This operator provides a constraint on the next moment in time.
  - $(\text{sad} \ \&\& \ \text{!rich}) \rightarrow X(\text{sad})$
  - $((x==0) \ \&\& \ (\text{add3})) \rightarrow X(x == 3)$
- **F (finally)** - At some point in the future, this property will be true.
  - $(\text{funny} \ \&\& \ \text{ownCamera}) \rightarrow F(\text{famous})$
  - $\text{sad} \rightarrow F(\text{happy})$
  - $\text{send} \rightarrow F(\text{receive})$

# LTL Examples

- G (globally) - This property must always be true.
  - winLottery  $\rightarrow$  G(rich)
- U (until) - One property must be true until the second becomes true.
  - startLecture  $\rightarrow$  (talk U endLecture)
  - born  $\rightarrow$  (alive U dead)
  - request  $\rightarrow$  (!reply U acknowledgement)

# More LTL Examples

- $G(\text{requested} \rightarrow F(\text{received}))$
- $G(\text{received} \rightarrow X(\text{processed}))$
- $G(\text{processed} \rightarrow F(G(\text{done})))$
- If the above are true, can this be true?
  - $G(\text{requested}) \ \&\& \ G(\text{!done})$

# Computation Tree Logic Formulae

Combines quantifiers over all paths and path-specific quantifiers:

<b>A (all)</b>	A hunger	Starting from the current state, I must be hungry on all paths.
<b>E (exists)</b>	E hunger	There must be some path, starting from the current state, where I am hungry.

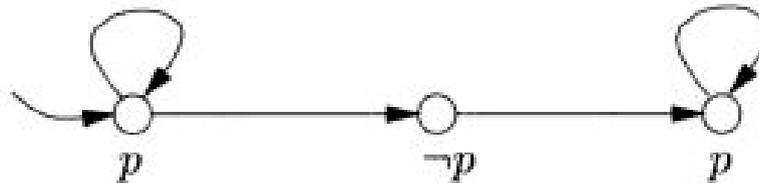
<b>X (next)</b>	X hunger	In the next state on this path, I will be hungry.
<b>G (globally)</b>	G hunger	In all future states on this path, I will be hungry.
<b>F (finally)</b>	F hunger	Eventually on this path, there will be a state where I am hungry.
<b>U (until)</b>	hunger U burger	On this path, I will be hungry until I start to eat a burger. (I must eventually eat a burger)
<b>W (weak until)</b>	hunger W burger	On this path, I will be hungry until I start to eat a burger. (There is no guarantee that I eat a burger)

# CTL Examples

- chocolate = “I like chocolate.”
- warm = “It is warm outside.”
- AG chocolate
- EF chocolate
- AF (EG chocolate)
- EG (AF chocolate)
- AG (chocolate U warm)
- EF ((EX chocolate) U (AG warm))

# Examples

- It is always possible to reach a state where we can reset.
  - **AG (EF reset)**
  - Is the LTL formula **G (F reset)** the same expression?
- Eventually, the system will reach a good state and remain there.
  - **F (G good)**
  - Is the CTL formula **AF (AG good)** the same?

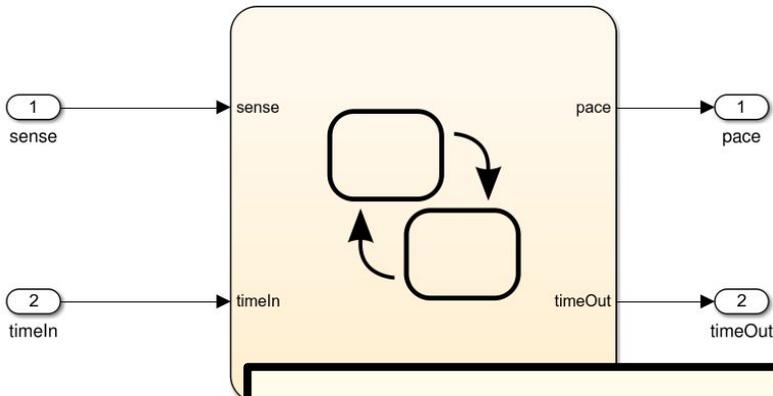


# Proving Properties Over Models

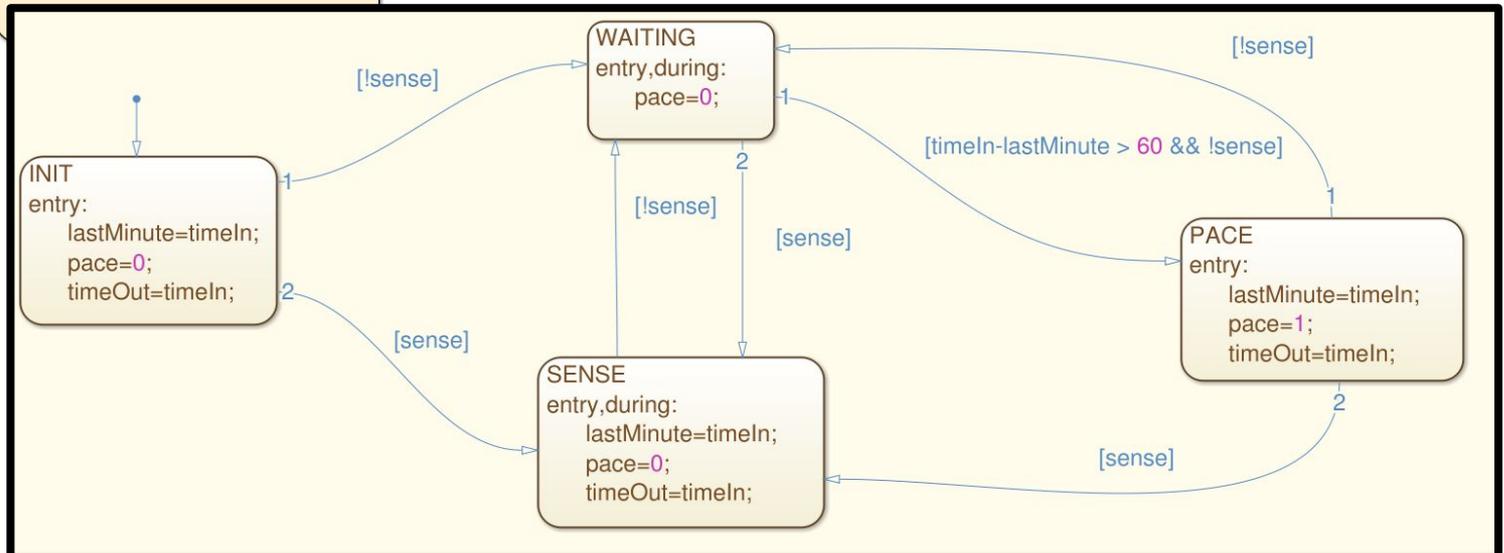
# Building Models

- Many different modeling languages.
- Most verification tools use their own language.
- Conceptually, most map to state machines.
  - Define a list of variables.
  - Describe how their values are calculated.
  - Each “time step”, recalculate the values of these variables.
  - The state is the current set of values for all variables.

# Creating Models - Graphical



- Most common industrial framework: Stateflow



# Creating Models - Written Language

```
MODULE main
VAR
request: boolean;
status: {ready, busy};
ASSIGN
init(status) := ready;
next(status) :=
case
    status=ready & request: busy;
    status=ready & !request : ready;
    TRUE: {ready, busy};
esac;
```

- NuSMV modeling language
- Part of a framework for model analysis.

# Proving Properties

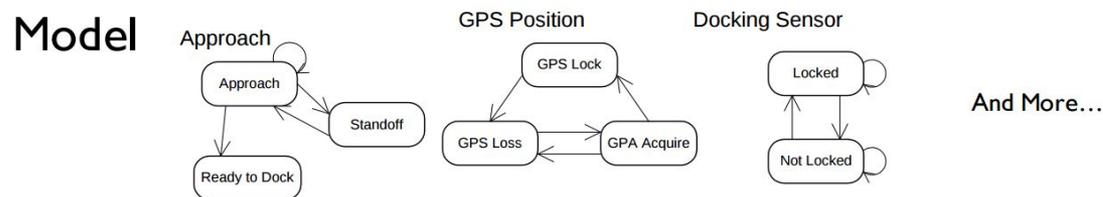
- To perform verification, we take properties and exhaustively search the state space of the model for violations.
- Violations give us counter-examples
  - A path that demonstrates how the property has been violated.
- Implications:
  - Property is incorrect.
  - Model does not reflect expected behavior.
  - Real issue found in the system being designed.

# Test Generation from FS Verification

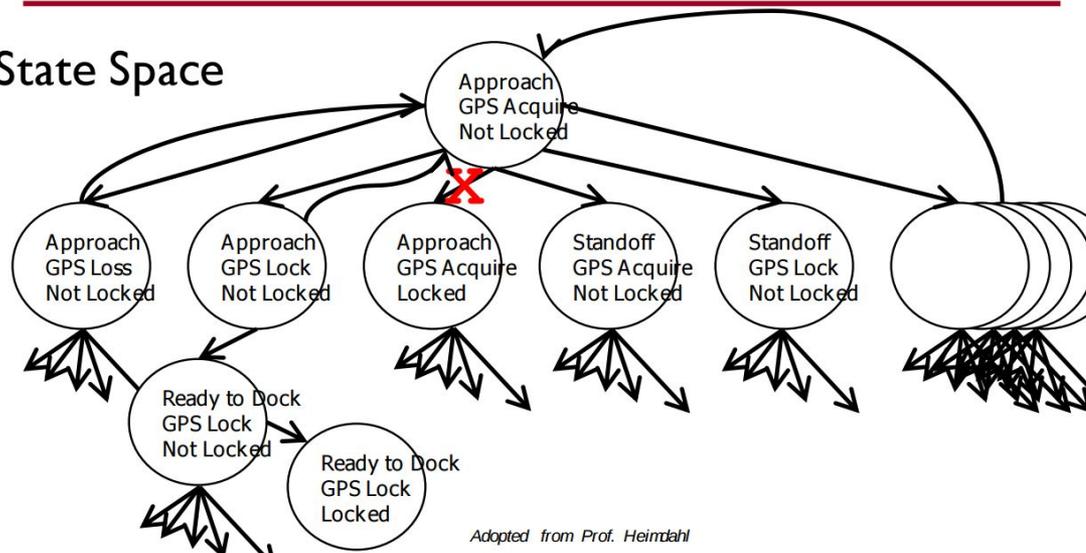
- We can also take properties and **negate** them.
  - Called a “trap property” - we assert that a property can never be met.
- The counter-example shows one way the property can be met.
- This can be used as a test for the real system - to demonstrate that the final system meets its specification.

# Exhaustive Search

- Algorithms exhaustively comb through the possible execution paths through the model.
- Major limitation - state space explosion.



## State Space



# Exhaustive Search - Dining Philosophers

- Problem -  $X$  philosophers sit at a table with  $Y$  forks between them. Philosophers may think or eat. When they eat, they need two forks.
- Goal is to avoid deadlock - a state where no progress is possible.
  - 5 philosophers/forks - deadlock after exploring 145 states
  - 10 philosophers/forks - deadlock after exploring 18,313 states
  - 15 philosophers/forks - deadlock after exploring 148,897 states
  - 9 philosophers/10 forks - deadlock found after exploring 404,796 states

# Search Based on SAT

- Express properties as conjunctive normal form expressions:
  - $f = (!x_2 \vee x_5) \wedge (x_1 \vee !x_3 \vee x_4) \wedge (x_4 \vee !x_5) \wedge (x_1 \vee x_2)$
- Examine reachable states and choose a transition based on how it affects the CNF expression.
  - If we want  $x_2$  to be false, choose a transition that imposes that change.
- Continue until CNF expression is satisfied.

# Branch & Bound Algorithm

- Set a variable to a particular value (true/false).
- Apply that value to the CNF expression.
- See whether that value satisfies all of the clauses that it appears in.
  - If so, assign a value to the next variable.
  - If not, backtrack (bound) and apply the other value.
- Prune branches of the boolean decision tree as values are applied.

# Branch & Bound Algorithm

$$f = (!x_2 \ || \ x_5) \ \&\& \ (x_1 \ || \ !x_3 \ || \ x_4) \ \&\& \ (x_4 \ || \ !x_5) \ \&\& \ (x_1 \ || \ x_2)$$

## 1. Set $x_1$ to false.

$$f = (!x_2 \ || \ x_5) \ \&\& \ (0 \ || \ !x_3 \ || \ x_4) \ \&\& \ (x_4 \ || \ !x_5) \ \&\& \ (0 \ || \ x_2)$$

## 2. Set $x_2$ to false.

$$f = (1 \ || \ x_5) \ \&\& \ (0 \ || \ !x_3 \ || \ x_4) \ \&\& \ (x_4 \ || \ !x_5) \ \&\& \ (0 \ || \ 0)$$

## 3. Backtrack and set $x_1$ to true.

$$f = (0 \ || \ x_5) \ \&\& \ (0 \ || \ !x_3 \ || \ x_4) \ \&\& \ (x_4 \ || \ !x_5) \ \&\& \ (0 \ || \ 1)$$

# DPLL Algorithm

- Set a variable to a particular value (true/false).
- Apply that value to the CNF expression.
- If the value satisfies a clause, that clause is removed from the formula.
- If the variable is negated, but does not satisfy a clause, then the variable is removed from that clause.
- Repeat until a solution is found.

# DPLL Algorithm

$f = (!x2 \ || \ x5) \ \&\& \ (x1 \ || \ !x3 \ || \ x4) \ \&\& \ (x4 \ || \ ! \ x5) \ \&\& \ (x1 \ || \ x2)$

## 1. Set x2 to false.

$f = (1 \ || \ x5) \ \&\& \ (x1 \ || \ !x3 \ || \ x4) \ \&\& \ (x4 \ || \ ! \ x5) \ \&\& \ (x1 \ || \ 0)$

$f = (x1 \ || \ !x3 \ || \ x4) \ \&\& \ (x4 \ || \ ! \ x5) \ \&\& \ (x1)$

## 2. Set x1 to true.

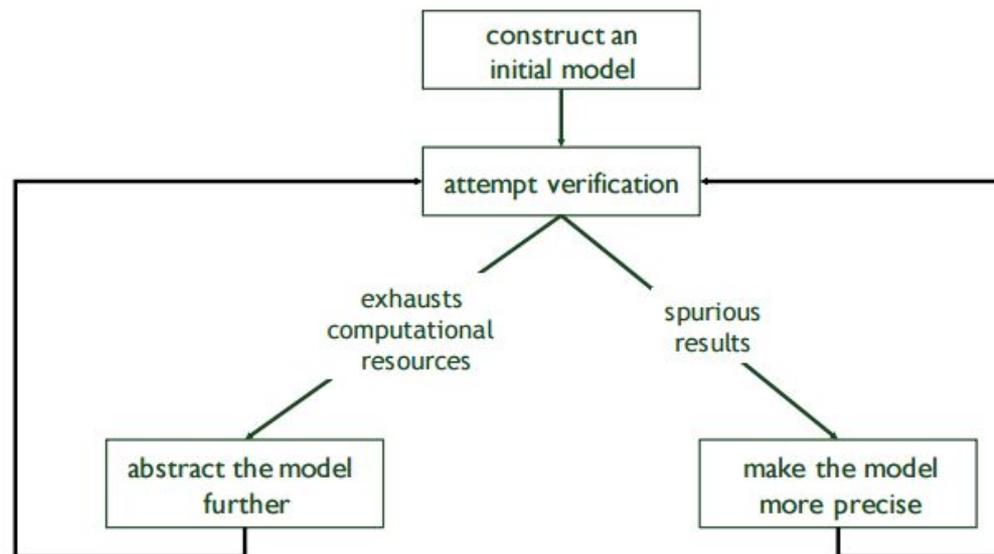
$f = (1 \ || \ !x3 \ || \ x4) \ \&\& \ (x4 \ || \ ! \ x5) \ \&\& \ (1)$

$f = (x4 \ || \ ! \ x5)$

## 3. Set x4 to false, then x5 to false.

# Model Refinement

- Models have to balance precision with efficiency.
- Abstractions that are too simple may introduce spurious failure paths that may not be in the real system.
- Models that are too complex may render model checking infeasible due to resource exhaustion.



# Intensional Models

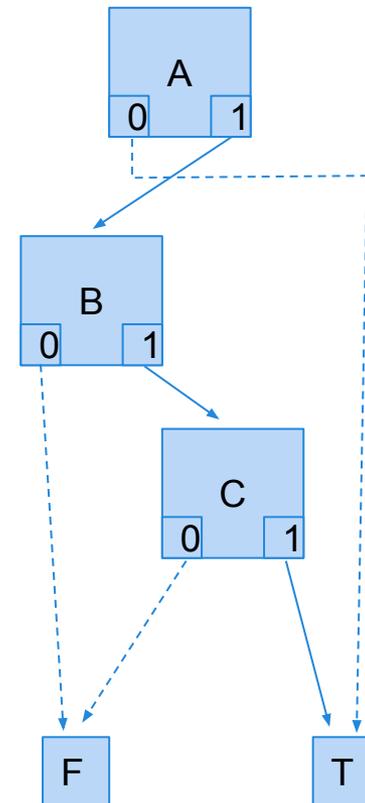
- State space can be limited by replacing *extensional* representations with *intensional* representations
  - A positive even integer  $< 20$ :
    - Extensional:  $\{2, 4, 6, 8, 10, 12, 14, 16, 18\}$ 
      - (All concrete values)
    - Intensional:  $\{x \in \mathbb{N} \mid x \bmod 2 = 0 \wedge 0 < x < 20\}$ 
      - (Symbolic representation)
      - Equation called the *characteristic function*
        - A predicate true for all elements in the set of values and false otherwise.

# Ordered Binary Decision Diagrams

- We can represent whether or not there is a transition between two states using a characteristic function.
  - $f(m,n) = \text{true}$  if there is a transition from  $m$  to  $n$ .
- OBDDs are a data structure representing the calculation of a binary function.
  - Such as a characteristic function.
  - Can be used to represent a subset of the state space.

# OBDD Example

- $\neg a \vee (b \wedge c)$ 
  - Can be thought of as a function:  $f(a,b,c)$
  - Returns true if the property is satisfied, false if not.

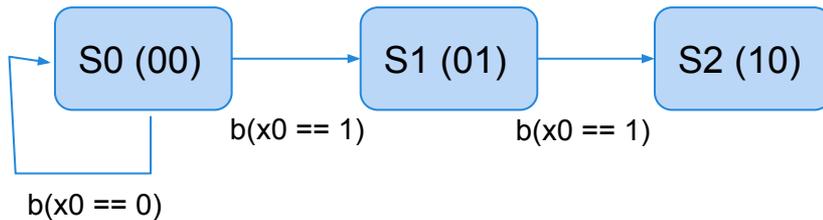


# Ordered Binary Decision Diagrams

- Built by iteratively expanding the set of states reachable in  $k+1$  steps.
  - Stabilizes when the number of transitions that can occur in the next step are already included.
- Most basic form - what states can we reach from the current state in  $n$  transitions?
- Often, merged with specification properties:
  - The set of transitions leading to a violation of the property.
  - If that set is empty, the property is verified.
    - Symbolic model checking.

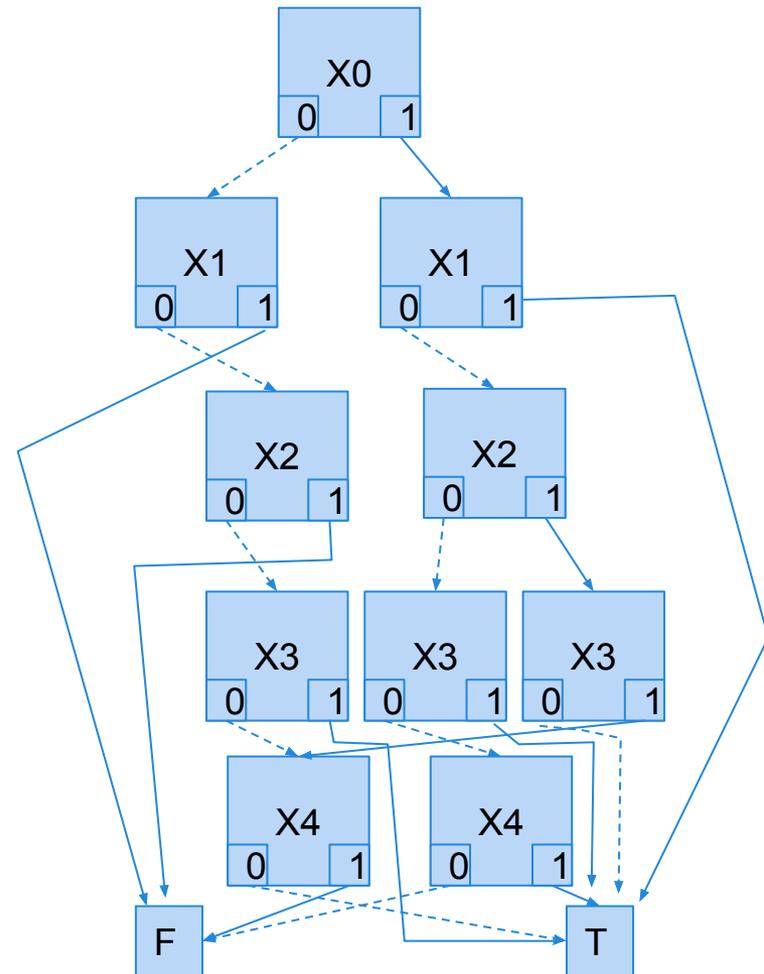
# Building OBDDs

- Assign each state and symbol a boolean label.



- Encode transitions as tuples (sym, from, to)

X0	X1X2	X3X4
0	00	00
1	00	01
1	01	10
sym	from state	to state



# Benefits of OBDDs

- OBDDs allow us to represent *sets of states* symbolically.
  - Rather than reasoning over the entire state space, we can reason over a small representation of a set of states (the boolean characteristic function).
- Allows verification of much larger models than explicit model checking.
  - As long as we *can* represent states with such a function.
  - Best when there is a large degree of regularity in the state space.

# We Have Learned

- We can perform verification by creating models of the system and proving that the specification properties hold over the model.
- To do so, we must express specifications as sets of logical formulae written in a temporal logic.
- Finite state verification exhaustively searches the state space for violations of properties.

# We Have Learned

- By performing this process, we can gain confidence that the system will meet the specifications.
  - We can even generate test cases from the model to help demonstrate that properties still hold over the final system.

# Next Time

- Symbolic execution and proof of properties
- Reading: Chapter 7
  
- Homework:
  - Reading assignment 3 is out.
    - Steven P. Miller, Alan C. Tribble, Michael W. Whalen, and Mats P.E. Heimdahl. Proving the Shalls: Early Validation of Requirements Through Formal Methods
    - Due April 4th.
  - Assignment 3 due tonight.