# Symbolic Execution and Proof of Properties

CSCE 747 - Lecture 22 - 03/30/2017

# Symbolic Execution

- Process of building predicates that describe which execution paths will be taken and their effect on program state.
  - Determines the conditions under which a path can be taken.
  - Identifies infeasible paths and paths that can be taken when they shouldn't.
  - Can be used to generate tests targeted at particular paths in the system.

# Symbolic Execution

- Bridge between complex program behavior and analyzable logical structures.
  - Enables complex analyses of programs through abstraction to a model of execution.
  - Allows proof of properties over small critical subsystems.
  - Allows formal verification of critical properties resistant to testing.
  - Allows formal verification of logical designs before code is written.

# What is Symbolic Execution?

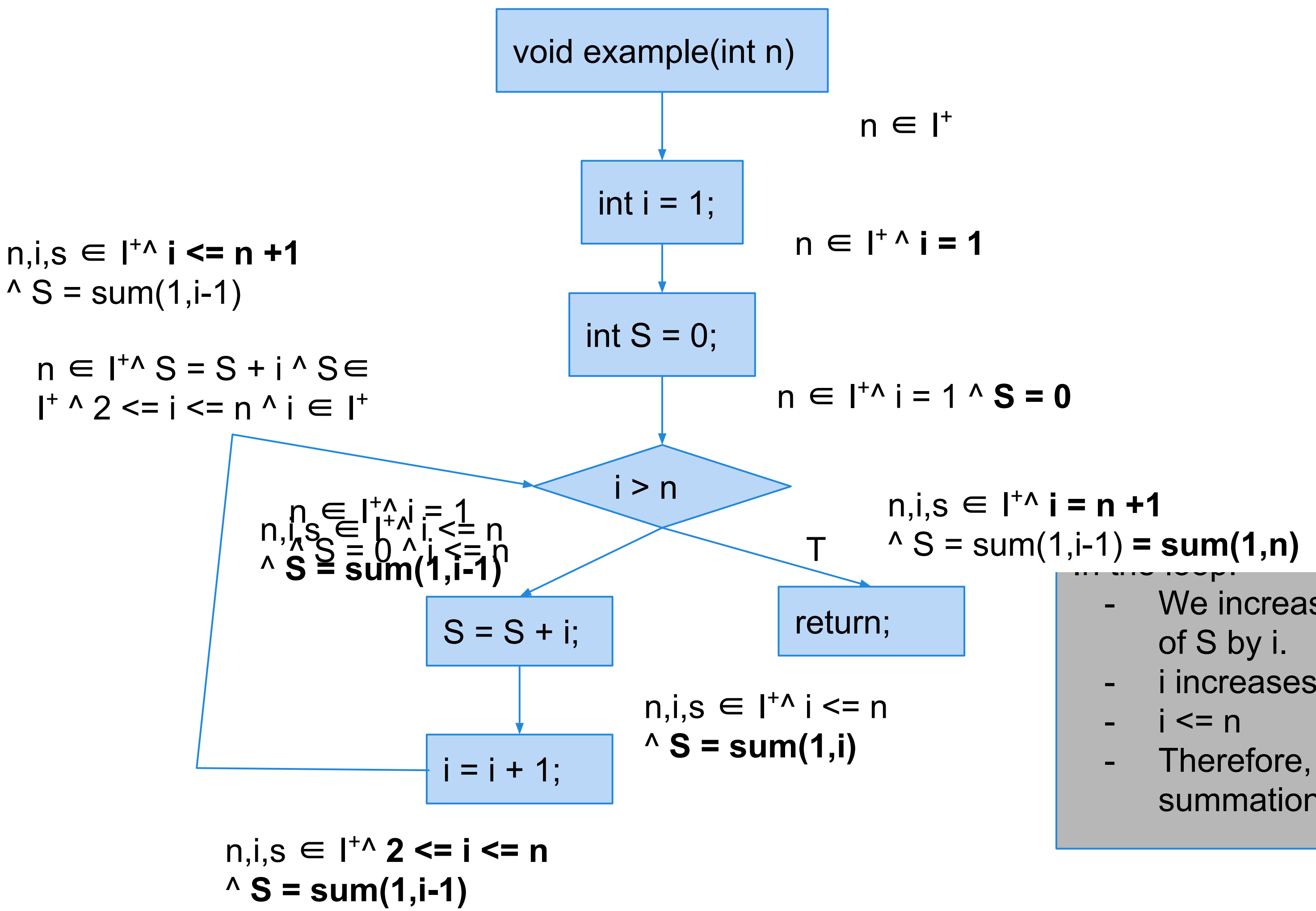
## Program Execution

- Execute the program with actual values.
- Statements compute new values for variables.
- Program state can be characterized by the values of variables.

## Symbolic Execution

- Execute the program with symbolic values
- Statements compute new symbolic expressions
- Program state can be characterized by predicates made of symbolic expressions

# Assigning Meaning to Programs




In the loop:

- We increase the value of S by i.
- i increases by 1.
- i <= n
- Therefore, S is a summation over 1 to i

# Binary Search

```
char *binarySearch( char *key, char *dictKeys[], char *dictValues[], int dictSize ) {
     int low = 0;
     int high = dictSize – 1;
     int mid, comparison;
     while (high >= low) {
          mid = (high + low) / 2;
          comparison = strcmp( dictKeys[mid], key );
          if (comparison < 0) {
               low = mid + 1;
          } else if ( comparison > 0 ) {
               high = mid - 1;
          } else {
               return dictValues[mid];
          }
     }
     return 0;
}
```

# Effect of Executing a Statement

```
mid = (low + high) / 2;
```

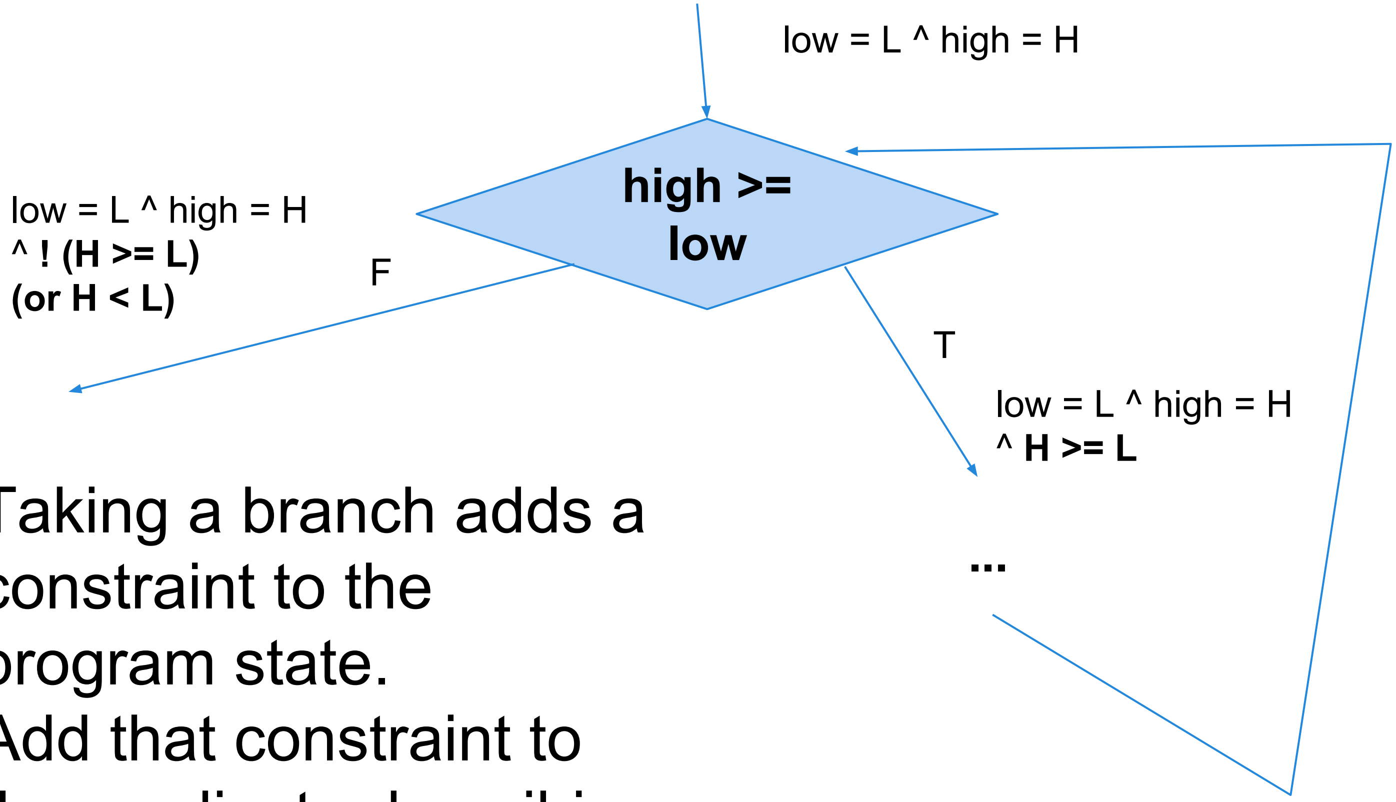
## Concrete Values

- Before:
  - low = 8 ^ high = 13
- After:
  - low = 8 ^ high = 13 ^ mid = 10

## Symbolic Values

- Before:
  - low = L ^ high = H
- After:
  - low = L ^ high = H ^ mid = (L + H) / 2

# Dealing with Branches




- Taking a branch adds a constraint to the program state.
- Add that constraint to the predicate describing the state.

# Symbolic Execution

- “Satisfying the predicate” can mean finding concrete values that make it evaluate to true.
    - This is a test case forcing the program to take a path. If no values can be found, then this is an infeasible path.
- If there are a finite number of paths in a program, a symbolic executor can trace each and obtain predicates characterizing each one.

# Summary Information

- Symbolic representation of state can easily grow too complex to use.
  - And potentially an infinite number of paths.
- Can *simplify* the property we are checking:
  - P characterizes a state.
  - P => W
    - W is a simpler predicate than P.
  - We can use W instead of P.
    - W is a *summary* of P.

# Example: Summary Information

```
mid = (low + high) / 2;
```

## Symbolic Values

- Before:
  - low = L ^ high = H
- After:
  - low = L ^ high = H ^ mid = M ^ H >= M >= L

# Assertions

- Weaker predicate based on what must be true for the program to execute correctly.
  - Cannot be derived automatically.
- Also known as an **assertion**.
  - A predicate stating what *should* be true at a particular point in program execution.
- Making an assertion marks our intention to verify that the predicate is true.
  - and that it is acceptable to replace part of the state with that property.

# Effect of Weakening

- Required at times to make symbolic execution possible for complex programs.
- That predicate is no longer sufficient to find input that forces execution along that path.
  - Satisfying that predicate is *necessary but not sufficient* to exercise the path.
  - Showing that the predicate cannot be satisfied still shows that the path is infeasible.

# Working with Loops

- Number of paths is infinite in the presence of loops.
- To reason with loops in symbolic execution:
    - Use a summary (assertion) to describes the program state when control reaches the loop.
        - Called a *loop invariant*.
    - Does not change based on the number of iterations.
    - When execution reaches the invariant, we check that the loop invariant is true at that point.

# Verifying Correctness

- Choose a program segment.
  - At the beginning of that segment, place an assertion that must be true (a pre-condition).
  - At the end, place another assertion that must be true (a post-condition).
- Every program path is a sequence of segments from one assertion to the next.
- Verification = ensuring that any possible sequence of segments is logically valid with pre/post-conditions.

# Example - Binary Search

```
char *binarySearch( char *key, char *dictKeys[], char *dictValues[], int dictSize ) {
    int low = 0;
    int high = dictSize - 1;
    int mid, comparison;
    while (high >= low) {
        mid = (high + low) /
        comparison = strcmp( dictKeys[mid], key );
        if (comparison < 0) {
            low = mid + 1;
        } else if ( comparison > 0 ) {
            high = mid - 1;
        } else {
            return dictValues[mid];
        }
    }
    return 0;
}
```

**pre-condition:** ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]

- If the client obeys the pre-condition, the program will obey the post-condition.

**loop invariant:** ∀ i, 0 < i < size: dictKeys[i] = key => low <= i < high

- True when we reach the loop.
- True at beginning of each loop cycle.
- True after the end of the loop.
- Symbolic execution begins with the invariant and determines that it is true again following the path.
- The pre-condition must remain true as well.
    - The full loop invariant includes the pre-condition.

# Example - Binary Search

PC ^ low = M+ 1 ^ high = H ^ mid = M ^
∀ k, 0 < k < size: dictKeys[k] = key => **M+1** <= k < H

bindings ^ PC ^ **mid = M** ^ LI ^ **H >= M >= L**

```
while (high >= low) {
    mid = (high + low) / 2;
    comparison = strcmp( dictKeys[mid], key );
    if (comparison < 0) {
        low = mid + 1;
    } else if ( comparison > 0 ) {
        high = mid - 1;
    } else {
        return dictValues[mid];
    }
}
```

bindings ^ PC ^ mid = M ^ LI ^ H >= M >= L ^ **dictKeys[M] <**

high = H ^ PC ^ mid = M ^ LI ^ H >= M >= L ^ dictKeys[M] < key ^ **low = M+1**

**pre-condition (PC):** ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]

**loop invariant (LI):** ∀ k, 0 < k < size: dictKeys[k] = key => L <= k < H

**bindings:** low = L ^ high = H

# Example - Binary Search

```
while (high >= low) {
    mid = (high + low) / 2;
    comparison = strcmp( dictKeys[mid], key );
    if (comparison < 0) {
        low = mid + 1;
    } else if ( comparison > 0 ) {
        high = mid - 1;
    } else {
        return dictValues[mid];
    }
}
```

PC ^ low = M+ 1 ^ high = H ^ mid = M ^
∀ k, 0 < k < size: dictKeys[k] = key => L <= k < **M-1**

bindings ^ PC ^ **mid = M** ^ LI ^ **H >= M >= L**

bindings ^ PC ^ mid = M ^ LI ^ H >= M >= L ^
**dictKeys[M] > key**

low = L ^ PC ^ mid = M ^ LI ^ H >= M >= L ^ dictKeys[M] <
key ^ **high = M-1**

**pre-condition (PC):** ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]

**loop invariant (LI):** ∀ k, 0 < k < size: dictKeys[k] = key => L <= k < H

**bindings:** low = L ^ high = H

# Example - Binary Search

bindings ^ PC ^ LI

bindings ^ PC ^ LI ^ **H >= L**

bindings ^ PC ^ **mid = M** ^ LI ^ **H >= M >= L**

```
while (high >= low) {
    mid = (high + low) / 2;
    comparison = strcmp( dictKeys[mid], key );
    if (comparison < 0) {
        low = mid + 1;
    } else if ( comparison > 0 ) {
        high = mid - 1;
    } else {
        return dictValues[mid];
    }
}
```

**Verify the contract of the procedure:**
*Returns corresponding value from dictValues for the key in dictKeys*, or null if key does not appear in dictKeys.

s=value ^ ∃ i, 0 <= i < size: dictKeys[i] = k ^ dictValues[i] = value

**pre-condition (PC):** ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]

**loop invariant (LI):** ∀ k, 0 < k < size: dictKeys[k] = key => L <= k < H

**bindings:** low = L ^ high = H

# Example - Binary Search

```
char *binarySearch( char *key, char
     int low = 0;
     int high = dictSize - 1;
     int mid, comparison;
     while (high >= low) {
          mid = (high + low) / 2;
          comparison = strcmp( dictKeys[mid], key );
          if (comparison < 0) {
               low = mid + 1;
          } else if ( comparison > 0 ) {
               high = mid - 1;
          } else {
               return dictValues[mid];
          }
     }
     return 0;
}
```

**pre-condition (PC):** ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]

**loop invariant (LI):** ∀ k, 0 < k < size: dictKeys[k] = key => L <= k < H

**bindings:** low = L ^ high = H

bindings ^ PC ^ LI ^ **L>H**

- Presence of the key implies L < H
- But, L > H
- Therefore, the key is not present.
- The post-condition is met.

**post-condition:** s=0 ^ ∄ a, 0 <= a < size : dictKeys[a] = key

**Verify the contract of the procedure:**
Returns corresponding value from dictValues for the key in dictKeys, or *null if key does not appear in dictKeys.*

# Activity

The loop body of the binary search can be modified to:

Demonstrate using symbolic execution that the path that traverses the false branch of all three statements is infeasible.

```
if (comparison < 0){
    low = mid + 1;
}
if (comparison > 0){
    high = mid -1;
}
if (comparison == 0){
    return dictValues[mid];
}
```

# Activity - Solution

```
if (comparison < 0){
     low = mid + 1;
}
if (comparison > 0){
     high = mid -1;
}
if (comparison == 0){
     return dictValues[mid];
}
```

low = L ^ high = H ^ mid = M ^ comparison = C ^ !(C<0)

low = **0)**

low = L ^ high = H ^ mid = M ^ comparison = C ^
**(!(C<0) ^ !(C>0) => (C=0))**

low = L ^ high = H ^ mid = M ^ comparison = C ^
(!(C<0) ^ !(C>0) => (C=0)) ^ **!(c=0)**

# Compositional Reasoning

- Programs can be structured and verified in a hierarchy of segments.
- Loop invariant is placed at beginning of the loop so we can compose facts about pieces of a program.
- Effect of a block is described as a *Hoare Triple*:
    - (|pre|) block (|post|)
    - If *pre* is satisfied at entry, then after executing *block*, *post* will be satisfied.

# Inference Rules

- Standard templates for reasoning with triples
- While Loops:

$$\frac{(|I \wedge C|)\ S\ (|I|)}{(I)\ \text{while}(C)\ \{\ S\ \}\ (|I \wedge !C|)}$$

- Formula on top line is the *premise*.
- Formula on the bottom line is the *conclusion*.
- If we can verify the premise, we can infer the conclusion.

# Inference Rules - While

- While Loops:

$$\frac{(|I \wedge C|)\ S\ (|I|)}{(|I|)\ \mathrm{while}(C)\ \{\ S\ \}\ (|I \wedge !C|)}$$

- Premise:
  - If invariant (I) and loop condition (C) are true before the loop, then after executing the loop body (S), I will still be true.
- Conclusion:
  - The loop takes the program from a state where I is true to a state where I is true and C is not.

# Inference Rules - If-Statement

$$\frac{(|P \wedge C|)\ \text{thenpart}\ (|Q|)\quad (|P \wedge !C)\ \text{elsepart}\ (|Q|)}{(|P|)\ \text{if(C) \{ thenpart \} else \{elsepart\}}\ (|Q|)}$$

- Premise:
  - If pre-condition (P) and if condition (C) are true, then after executing *thenpart* a postcondition (Q) will be true. If P is true and C is false, then after executing *elsepart*, Q is true.
- Conclusion:
  - The if-statement takes the program from a state where P is true to a state where Q is true.

# Compositional Reasoning

- Can compose proofs about small parts of the program into proofs about larger parts.
  - Inference rule for *while* lets us take a triple about the loop body and infer a triple about the whole loop.
- Summarize the effect of a block of code by a pre-condition and post-condition.
  - Can summarize the effect of the whole procedure in the same way.
  - Establish a *contract* for that block of code.

# Compositional Reasoning

- The contract of a procedure is:
  - Pre-condition: What the client is required to provide.
  - Post-condition: What the procedure promises to establish or return.
- Can use that contract whenever the procedure is called to verify input and results
- Binary Search:
  - (| ∀ i, j, 0 <= i < j < size: dictKeys[i] <= dictKeys[j]|)
  - s = binarySearch(k, dictKeys, dictValues, size)
  - (| (s=value ^ ∃ i, 0 <= i < size: dictKeys[i] = k ^ dictValues[i] = value) v s=0 ^ ∄ a, 0 <= a < size : dictKeys[a] = key)|)

# Activity 2 - Contract

- The following method calculates the sum of an array of floats.
- Write the pre- and post-conditions for this method.

```
float sum(int array[], int len) {
    float sum = 0.0;
    int i = 0;
    while (i < length) {
        sum = sum + array[i];
        i = i + 1;
    }
    return sum;
}
```

# Activity 2 - Contract

(|pre|) block (|post|)

(| len >= 0 ^ array.length = len|)

s = sum(array,len)

(|s = $\sum_{j=0}^{len}$ array[j]|)

```
float sum(int array[], int len) {
    float sum = 0.0;
    int i = 0;
    while (i < length) {
        sum = sum + array[i];
        i = i + 1;
    }
    return sum;
}
```

# Classes and Data Structures

- Classes often maintain data structures.
  - If a method is called on that structure, the responsibility for that structure's correctness belongs to the class, not the caller.
- Modular verification must obey modular design of the program.
  - Contract cannot reveal private details.

# Abstract Model of Data

- Data structure module provides a collection of methods with related specifications.
  - Specifications are contracts with clients.
  - Specify pre and post-conditions of an abstract model of the encapsulated data.
    - Dictionary:
      - Contracts in terms of <key,value> pairs.
      - Actual implementation could be a hashmap, sorted array, tree, etc.
      - Details of implementation hidden.
      - Reason over correctness of the abstraction.

# Structural Invariants

- Class must preserve properties over the (abstract) data structure it maintains.
  - If structure is sorted arrays, then the class must maintain the sorted order.
  - If structure is balanced search tree, then the class must keep the tree balanced.
- Called *structural invariants*.
  - Similar to loop invariant.
  - Must hold before method invocation and after return.

# Abstraction Function

- Behavior must reflect the abstract model.
- Need an *abstraction function* to map concrete states to abstract states.
  - For dictionary, map implementation to <key,value> pairs.
  - If the implementation is java.util.map, the contract for get(key) method:

    (|<key, value> ∈ ∅(dict)|)
    o = dict.get(k)
    (|o = value|)

# We Have Learned

- Symbolic execution is the process of establishing constraints on the values of variables as a particular path is taken.
  - Hand execution using symbols instead of concrete values. Rules governing *any* execution of a path.
  - Bridge from concrete execution of a complex program to mathematical logic structures that can be reasoned over.
  - Used to prove correctness of pieces of a program.

# We Have Learned

- To perform over loops, methods, and data structures, must establish contracts (pre and post-conditions) on pieces of the program.
    - Can then reason about combinations of these pieces, as correctness is proven over the program hierarchy.
    - Allows checkable specifications of intended behavior.

# Next Time

- Using symbolic execution in automated program analysis
- Reading: Ch. 19

- Homework:
  - Reading assignment 3 - due April 4th.
  - Assignment 4 - out now!